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Aubon Leslie Mabry

DIRECT DESIGN OF COLUMNS

SUBJECTED TO

AXIAL LOAD AND BENDING

A THESIS

Presented to

the Faculty of the Graduate Division

by

Aubon Leslie Mabry

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

in the School of Civil Engineering

Georgia Institute of Technology

June 1953

DIRECT DESIGN OF COLUMNS

SUBJECTED TO

AXIAL LOAD AND BENDING

Approved:

Date Approved by Chairman: June 8, 1953

ACKNOWLEDGEMENTS

I am especially indebted to Dr. Boris W. Boguslavsky, my thesis adviser, for his original idea and work in helping me to make the derivations, charts and tables contained in this treatise.

In addition I would like to express my thanks and appreciation to Dr. Saxe and Mr. Roy K. Jacobs, members of my reading committee, for their help and interest shown.

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SYMBOLS AND NOTATION

A : a group of terms in formula (16a)

A_g : gross area of concrete section

A_s : total area of vertical column bars

B : a group of terms in formula (16a)

C_c : resultant of concrete compressive stresses

C_s : compressive stresses in vertical steel

D : $t/2R_g^2$; t is total depth of column; R_g is the radius of gyration

M : external moment (kip-ft.)

N : external load

R : radius to outside surface of spiral column

T_s : tensile stress in vertical steel

V : collection of terms involving a' in formula (13)

W : collection of terms involving a' in formula (13)

a' : angle at center of a spiral column between extreme concrete fiber and intersection of neutral axis with outside surface

b : width of tied column

d : effective depth of flectural members

d' : depth from extreme fiber to center of reinforcement in direction of bending

e : eccentricity measured from gravity axis (in.)

SYMBOLS AND NOTATION (Cont'd)

f_c : compressive stress in extreme fiber

f'_c : ultimate compressive stress in concrete

f_p : allowable stress in eccentrically loaded columns

f_s : stress in vertical column reinforcement

f_a : average stress in axially loaded columns

g : ratio of diameter of circle (gt) through bar centerlines, or distance (gt) between bar centerlines at opposite forces of tied column to over-all dimension (t)

k : ratio of distance (kt) between extreme compressive fiber and neutral axis to total depth (t)

n : ratio of modulus of elasticity of steel to that of concrete

p : ratio of vertical reinforcement to area of concrete section

r : radius of equivalent steel ring in spiral columns

t : over-all depth of column section

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CHAPTER I

INTRODUCTION

The problem undertaken here is to introduce a new and improved method whereby the design of reinforced concrete columns subjected to bending and direct stresses can be approached in a direct way. And further, the maximum use of the allowable concrete stresses can be realized so as to effect the most accurate and therefore most dependable and economical of designs. This is realized with a very great reduction in effort and tedium on the part of the designer, since by this new method he can, with the help of charts contained herein, obtain column sizes and percentages of steel required to meet precisely the given conditions of his design problem.

Graphs have been prepared here for square, or rectangular, tied columns and for round spiral columns. In both cases the ultimate compressive strength of concrete is fixed at 3000 pounds per square inch. No other strength concrete can be used here.

These graphs are based on trial and error solutions of a formula developed by elementary principles of statics and strength of materials combined with other empirical formulae developed through intensive testing by the American Concrete Institute and incorporated into their Building Code and also their Reinforced Concrete Design Handbook. These empirical formulae resulting from actual testing of columns are used as required by the code to determine the maximum safe allowable stresses where columns are subjected to combined bending and axial loads.

Fig. 1 below shows the stress distribution on a column section under bending and direct load.

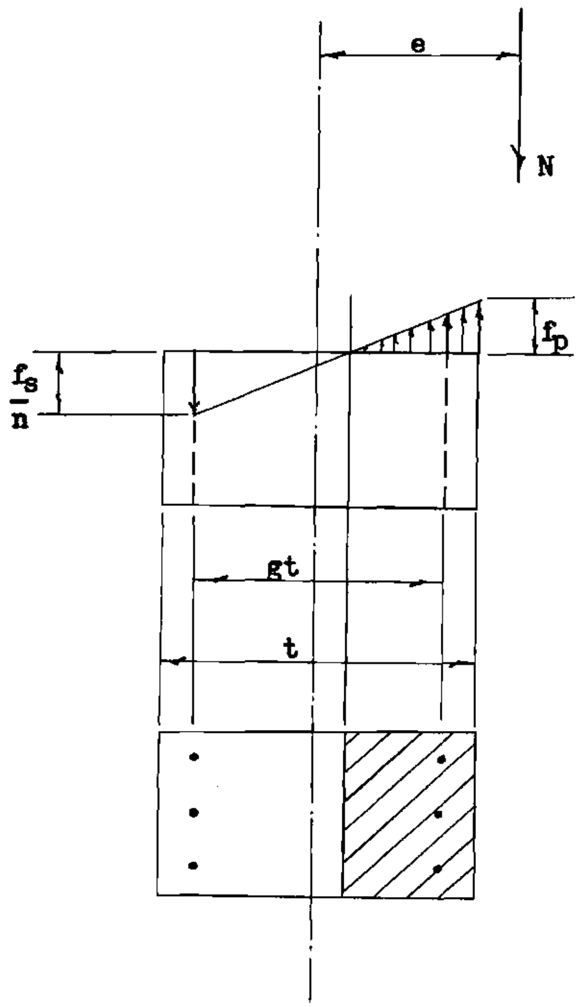

Fig. 1

Tied column section

The maximum allowable stress, f_p, for columns subjected to bending and axial loading is given in the Joint Committee and American Concrete Institute Building Code by the formula,

$$f_p = f_a \frac{1 + \frac{e}{t} D}{1 + \frac{f_a}{0.45 f_c} D \frac{e}{t}} \qquad (1)$$

where f_a is the average allowable stress on a column loaded axially, and

$$f_a = \frac{0.225\ f_c = f_s P}{1 + (n - 1)\ P} \qquad (2)$$

for spiral columns and 0.8 of this value for tied columns.

Formerly, column design was done by assuming a column size and per cent steel, computing f_p by the above formulae, for that column, then with f_p determined for that size and per cent steel, the column was finally checked to determine if it would carry the load and bending moment designed for. If not, then another size was assumed and this process repeated, until a design was reached in which f_p was a little less than the allowable value. This means that on every large building design hundreds and perhaps thousands of repetitious computations had to be made by the design engineer. Columns, as they occur today in continuous structures including rigid frames, are almost never axially loaded, although in the past they have been designed as axially loaded members. For years a need has been felt by designers for tables or graphs to aid them and therefore save many hours of time and labor in both design and investigation of columns.

A notable step in that direction was taken in 1940 by a Committee of the American Concrete Institute with A. J. Boase as author-chairman. Diagrams and tables were published which speeded up the application of formula (1), since by going into three different tables and making three different computations, f_a and f_p could be determined for an assumed column. This shortened the work considerably but left much to be desired because the design was still one of trial and error.

In 1947, I. E. Morris of Atlanta, Georgia, a member of the same committee, after an enormous amount of work published a booklet, Allowable Loads on Eccentrically Loaded Concrete Columns. This work has later been combined in the new Concrete Reinforcing Steel Institute Handbook. Here the designer will find tabulated the values of N, the value of columns for varying eccentricities of P, the allowable concentric load on the column for values of e, the eccentricity of load from gravity axis, in one inch increments from zero to 14 inches. The per cent of vertical steel, P, of the gross area A_g , of the column is also given together with number of bars of steel required and bar sizes.

The most recent step in the direction of brevity in column design was made by Professor Dewey M. McCain, Director of the Civil

Engineering Department, Mississippi State College. His interesting article on the use of the "Add-area Method" to shorten the process of column design appears in the August 1952 issue of Civil Engineering Magazine. By way of introduction, he mentions that the best solution to the problem would be a set of curves from which the values needed for a direct design could be had. This set of curves is essentially what is being offered in this treatise as the best solution to the problem of column design.

CHAPTER II

SETTING UP FORMULAE

Tied Columns.--Because of the many variables involved in the construction of the charts offered here as an aid in designing columns, some of the variables were fixed as follows:

$$f'_c = 3000 \text{ psi.}, \quad n = 10$$

$$f_s = 20{,}000 \text{ psi.}$$

$$d'/t = 0.5,\ 0.1,\ 0.15$$

From Fig. 2:

$$C_c = \tfrac{1}{2} f_p bkt$$

$$C_s = \tfrac{1}{2} f_p \left(1 - \frac{d'}{kt} \right) (n - 1)pbt$$

$$T_s = \tfrac{1}{2} f_p \cdot \frac{1 - k - \frac{d'}{t}}{k} \cdot npbt$$

Since e/t is the value now sought, the value, d, common to concrete beam design has been replaced by t, the total thickness of the column. The value of k for tied columns was determined in the following

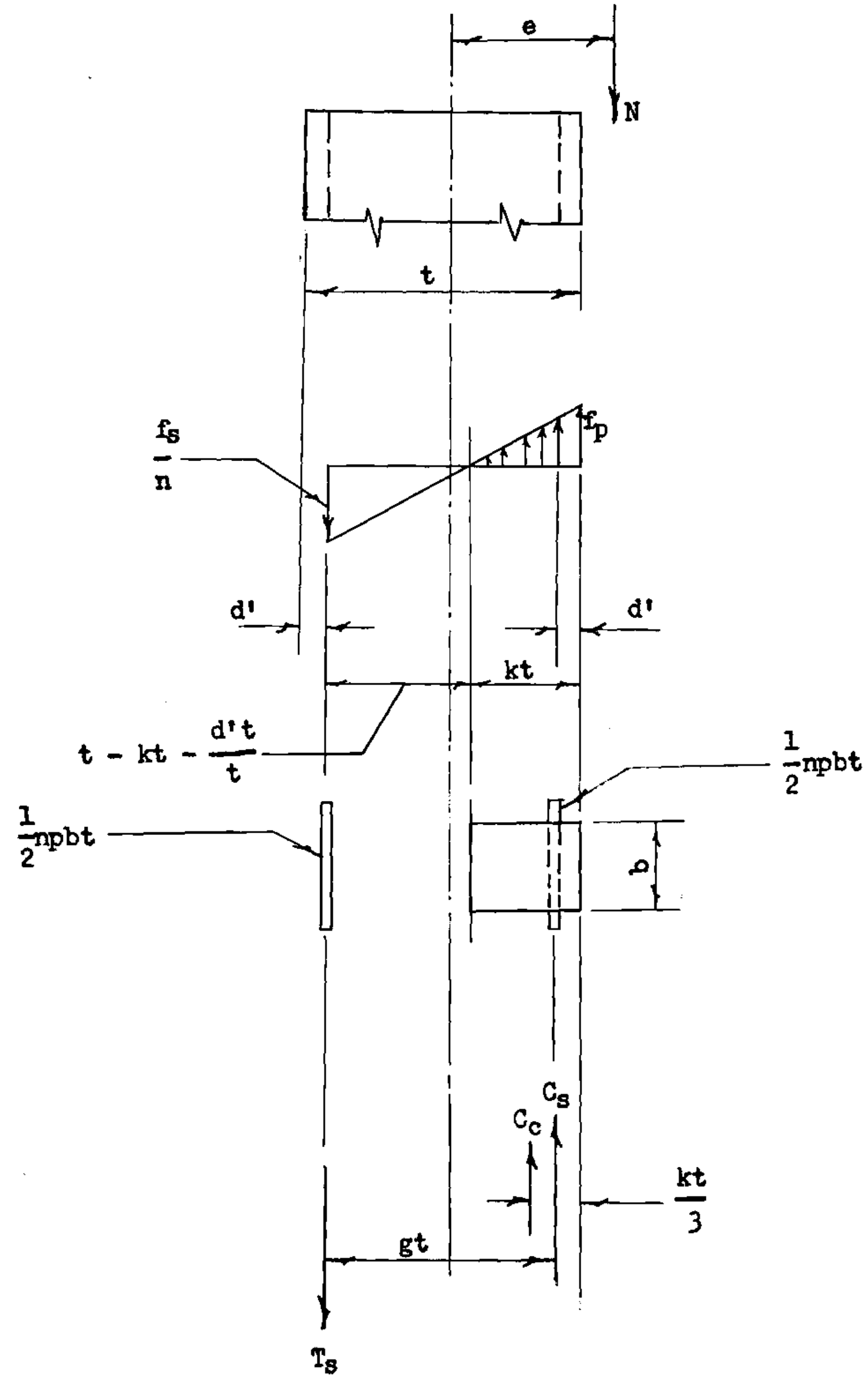

Fig. 2

Stress distribution of tied column under bending and direct load manner where:

$$t = d + d'$$

$$kt = k(d + d')$$

and, by similar triangles,

$$\frac{kt}{t - d'} = \frac{f_p}{f_p + \frac{f_s}{n}}$$

from which

$$k = \frac{f_p}{f_p + \frac{f_s}{n}} \left(1 - \frac{d'}{t}\right)$$

The normal load, N, applied at a distance, e, from the center of the column produces a moment, Ne, in addition to the normal load. This compressive load and the moment are balanced by the internal resisting stresses offered by both the reinforcing steel and concrete such that, the summation of vertical forces is zero, and

$$N = C_c + C_s - T$$

Then by substitution of the values of C_c, C_s, and T as given on page 6,

$$N = \tfrac{1}{2}f_p\, bkt + f_p \left(1 - \frac{d'}{kt}\right)(n - 1)\frac{p}{2}\, bt - f_p \frac{1 - k - \frac{d'}{t}}{k}\, n \frac{p}{2}\, bt$$

$$N = \tfrac{1}{2}f_p\, bt \left[k + \left(1 - \frac{d'}{kt}\right)(n - 1)\, p - np \frac{1 - k - \frac{d'}{t}}{k} \right]$$

$N = \frac{1}{2}f_p\, bt\; A$, where A is the quantity in brackets

$$bt = \frac{N}{\tfrac{1}{2}f_p\, A}, \text{ a design formula.} \qquad (3)$$

The values of $\frac{1}{2}f_p\, A$ are plotted in chart 1 against p so that once p is known, the size of the column is obtained from formula (3).

From Fig. 2 it is further seen that the internal resisting moment is:

$$Ne = \tfrac{1}{2}f_p\, bkt \left(\frac{t}{2} - \frac{kt}{3}\right) + \tfrac{1}{2}f_p \left(1 - \frac{d'}{kt}\right)(n - 1)\, pbt\; \frac{t}{2} - d'$$

$$+ \tfrac{1}{2}f_p\, npbt\; \frac{1 - k - \frac{d'}{t}}{k} \left(\frac{t}{2} - d'\right)$$

$$Ne = \tfrac{1}{2}f_p\ bt^2\left[\frac{k}{2} - \frac{k^2}{3} + \left(1 - \frac{d'}{kt}\right)(n - 1)p\left(\frac{1}{2} - \frac{d'}{t}\right) + \frac{1 - k - \frac{d'}{t}}{k}\ np\left(\frac{1}{2} - \frac{d'}{t}\right)\right] \tag{4}$$

$$\frac{Ne}{N} = \frac{\tfrac{1}{2}f_p\ bt^2B}{\tfrac{1}{2}f_p\ bt\ A}, \quad \text{and } e = \frac{bt}{A}$$

$$\text{Or,}\quad \frac{e}{t} = \frac{B}{A}$$

And $A =$ Col. 2 $+$ p (Col. 9) from Table 2.

$$\frac{e}{t} = \frac{\frac{k}{2} - \frac{k^2}{3} + \frac{1}{k}\left(k - \frac{d'}{t}\right)(n - 1)p\left(\frac{1}{2} - \frac{d'}{t}\right) + \frac{1}{k}\left(1 - k - \frac{d'}{t}\right) np\left(\frac{1}{2} - \frac{d'}{t}\right)}{k + \frac{1}{k}\left(k - \frac{d'}{t}\right)(n - 1)p - np\left(\frac{1}{k}\right)\left(1 - k - \frac{d'}{t}\right)}$$

Simplifying,

$$\frac{e}{t} = \frac{\dfrac{k}{2} - \dfrac{k^2}{3} + \dfrac{1}{k}\left(\dfrac{1}{2} - \dfrac{d'}{t}\right) p\left[n - k - \dfrac{d'}{t}(2n - 1)\right]}{k + \dfrac{p}{k}\left[k\,(2n - 1) - n + \dfrac{d'}{t}\right]} = \frac{B}{A} \tag{5}$$

Then from Fig. 2, and taking f_p as 1125 psi., Table 1,

$$k = \frac{1 - \dfrac{d'}{t}\; f_p}{f_p + \dfrac{f_s}{n}} = \frac{0.9 f_p}{f_p + 2000} = \frac{0.9(1125)}{1125 + 2000} = 0.324{,}000$$

where $f_s = 20{,}000$ psi., $n = 10$, and $d'/t = 0.10$ for example.

The allowable combined axial and bending stress, f_p, given in both the <u>Joint Committee Code 1940</u> and <u>The ACI Building Code 1951</u> for all types of reinforcement is:

$$f_p = f_a \frac{1 + \dfrac{e}{t} D}{1 + \dfrac{e}{t} \cdot \dfrac{f_a}{0.45 f'_c} D} \tag{6}$$

where $f_a = 0.8 \dfrac{0.225f'_c + f_s p}{1 + (n - 1)p}$

and $D = \dfrac{1 + (n - 1)p}{\frac{1}{6} + \frac{1}{2}(n - 1)pg^2}$

Always, t and gt are measured in the direction of the moment, Ne, and g is constant when d'/t is fixed.

$$gt + 2d' = t$$

and if for example $d'/t = 0.10$
then $d' = 0.1t$ so that,

$$g\cancel{t} + 0.2\cancel{t} = \cancel{t}$$

or, $g = 0.8$

Substituting the values f_a and D, in equation (5) it becomes:

$$f_p = 0.8 \frac{0.225f'_c + f_s p}{1 + (n - 1)p} \qquad (7)$$

$$\frac{1 + \dfrac{e}{t} \cdot \dfrac{1 + (n-1)p}{\dfrac{1}{6} + \dfrac{1}{2}(n-1)pg^2}}{1 + \dfrac{e}{t} \cdot \dfrac{0.8}{0.45f'_c} \cdot \dfrac{0.225f'_c + f_s p}{\dfrac{1}{6} + \dfrac{1}{2}(n-1)pg^2}}$$

Equation (7) cannot be solved directly because the value of e/t depends on the values of both f_p and p. Therefore, a trial and error solution is accomplished by setting up table 1 from which e/t, p, and $\frac{1}{2}f_pA$ found for the selected values of f_p used in the table by finding a trial value of p such that the computed value of f_p is very nearly equal to the corresponding table value of f_p. For example, f_p may be taken as 1125 psi. and a trial p of 0.01. Then, from Table 1:

$$\frac{e}{t} = \frac{\text{Col. } 5 + p\ (\text{Col. } 7)}{\text{Col. } 2 + p\ (\text{Col. } 9)} = 1.0699 \tag{8}$$

and using further the fixed values of n - 10, $f'_c = 3000$, $f_s = 20{,}000$, and d'/t - 0.10, f_p is determined as follows:

$$f_p = 0.8 \quad \frac{0.225(3000) + 20{,}000(0.01)}{1 + (10 - 1)\ (0.01)}$$

$$\frac{1 + 1.0699 \; \dfrac{1 + 9\ (0.01)}{\frac{1}{6} + \frac{1}{2}\ (9)\ (0.01)\ (0.8)^2}}{1 + 1.0699 \; \dfrac{0.8(0.225)\ (3000) + 20{,}000(0.01)}{0.45(3000)\ \ \frac{1}{6} + \frac{1}{2}\ (9)\ (0.01)\ (0.8)^2}}$$

$f_p = 1165.6$ psi.

This is higher than the table value of 1125 psi. upon which the value of k and therefore values of Columns 2, 5, 7, and 9 depend. Therefore, the values of p, e/t, $\frac{1}{2}f_pA$ based on these computations would be in error.

Try a steel ratio, p, of 0.0777. Then, e/t from table and formula (8) is 0.861 and,

$$f_p = 0.8 \cdot \frac{830}{1.0698} \cdot \frac{1 + 0.861 \; \dfrac{1.06975}{0.1890}}{1 + 0.861 \; \dfrac{664}{1350(0.1890)}}$$

$f_p = 1124.99.$

This is very close and the values of p, $\frac{1}{2}f_pA$, and e/t are recorded in Table 1 to be later plotted.

This process of trial and error computations was repeated for values of f_p in 25 psi increments from 1125 to 1275 pounds per square inch. The curves e/t vs p and $\frac{1}{2}f_pA$ vs p were plotted on the same chart for three values of d'/t: 0.05, 0.10, and 0.15. Assuming that one and one-half inches is satisfactory cover for the steel bars the center of a one inch bar would be two inches from the outside surface of the column, or d' is two inches.

This allows for interpolation between the values of d'/t from 0.05 to 0.15 which corresponds to column diameters from 14 inches to 40 inches.

Round Spiral Columns.--The formula for f_p in the case of round spiral columns is very similar to the equation (7), page 12. Its derivation is much the same, with only minor differences arising from the circular instead of the rectangular section. However, the 1940 Joint Committee and ACI Code for 1951 specify that the value of f_a shall be:

$$f_a = \frac{0.225f'_c + f_s\,p}{1 + (n - 1)p} \tag{9}$$

and the value of D from the Reinforced Concrete Design Handbook changes for round sections to:

$$D = \frac{1 + (n - 1)p}{\frac{1}{8} + \frac{1}{4}(n - 1)pg} \tag{10}$$

The value of e/t is more difficult to obtain however. The derivation of the formula for $\frac{e}{R} = 2\frac{e}{t}$ appears in Reinforced Concrete Structures by Professor Dean Peabody.[1] It follows that where kt = 2kR and,

$$W = 12a_1 - 3 \sin 4a_1 - 32 \cos a_1 \sin^3 a_1 \tag{11}$$

$$V = 2 \sin^2 a_1 + 3 \cos a_1 (\sin a_1 \cos a_1 - a_1) \tag{12}$$

$$\frac{e}{t} = \frac{W + 24\pi np \left(\frac{r}{R}\right)^2}{32(V - 3\pi np \cos a_1)} \tag{13}$$

also, $2R = t = 2r + 2d'$

and taking $d' = 0.1t$

$$gt + 0.2t = t$$

$$g = 0.8$$

$$\frac{r}{R} = \frac{\frac{gt}{2}}{\frac{t}{2}} = g$$

---

[1] Peabody, Dean Jr., Reinforced Concrete Structures. New York: John Wiley and Sons, Inc., 1936, pp. 251-252.

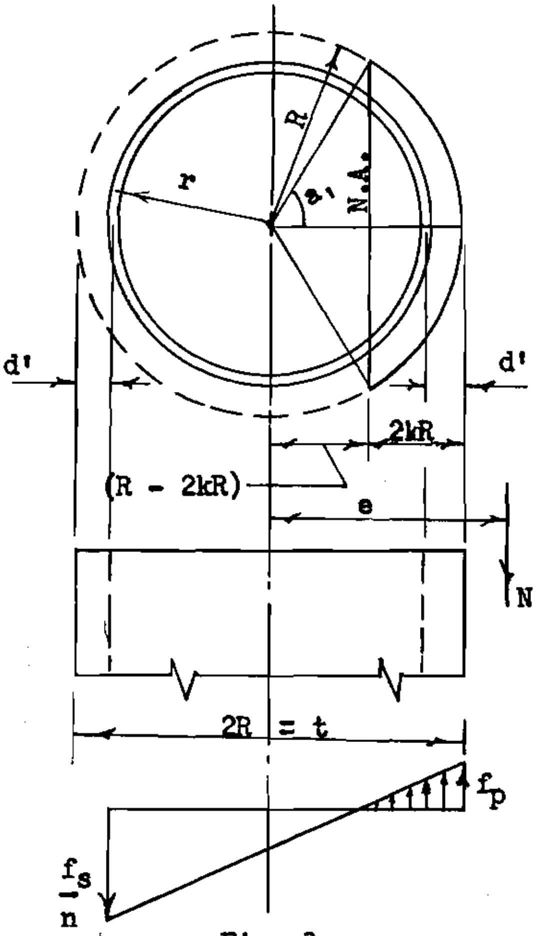

Fig. 3

Stress distribution of spiral column under bending and direct load

The formula for f_p for round spiral columns now becomes:

$$f_p = \left[\frac{0.225f'_c + f_s p}{1 + (n-1)p}\right] \left[\frac{1 + \frac{e}{t} \cdot \frac{1 + (n-1)p}{\frac{1}{8} + \frac{1}{4}(n-1)pg^2}}{1 + \frac{e}{t} \cdot \frac{0.225f'_c + f_s p}{0.45f'_c} \cdot \frac{1}{\frac{1}{8} + \frac{1}{4}(n-1)pg^2}}\right] \tag{14}$$

Tables 4, 5 and 6 have been set up and as with tied columns in Tables 1, 2 and 3, f_p was taken in 25 psi. increments from 1200 to 1325 psi. Also, k was computed from formula (5) for these values as in Tables 1, 2 and 3.

Note that using $np\pi R^2$ for the equivalent area of steel instead of the more accurate value, $(n - 1)p\pi R^2$ assumes p per cent more concrete than is actually there. This slight error is of minor importance however, since it varies from only 0.6 to 2.9 per cent for the entire range of values covered in the chart for round columns. This slight deviation permits identical thickness of the steel ring in both tension and compression areas of the section, greatly simplifying the formula and computations. Morever, since no allowance is made for any tension in concrete, only that part of the column section in compression is affected by this change.

An example to illustrate the use of formula (14) follows:

Taking $$\frac{d'}{t} = 0.10$$

and $$g = \frac{r}{R} = 0.8$$

$$f_s = 20{,}000$$

$$f'_c = 3{,}000$$

$$n = 10$$

A reasonable value of the allowable stress in a column made up of the above strength steel and concrete is selected--say,

$$f_p \text{ (allowable)} = 1200 \text{ psi.}$$

It now remains to vary p, the per cent steel until the right side of equation (14) is equal to 1200. Notice that line 1 of Tables 4, 5 and 6 are set up with f_p taken as 1200 psi. This fixes the value of k and all the other values in line 1 except the design factors, from column 20 onward. These can be picked out as in the case of p and e/t from the correctly chosen trial computation. The others are readily computed and are self-explanatory from the column headings.

The trial e/t is determined from formulas (11), (12), (13) and (14) and using the fixed value of k for 1200 psi. to find the angle a_1.

The correct computation for f_p chosen at 1200 psi. follows:

Trying p = 0.00662

$$1 + 9p = 1.05958$$

$$24\pi npg^2 = 3.194{,}468$$

$$p(\text{Col. } 19) = 6.488{,}772$$

$$0.225f'_c + 20{,}000\, p = 807.4$$

$$\frac{1}{8} + \frac{1}{4}\,(9)\, pg^2 = 0.134{,}576$$

From Table 2,

$$\frac{e}{t} = \frac{\text{Col. } 17 + 3.194{,}468}{\text{Col. } 18 - p(\text{Col. } 19)}$$

$$\frac{e}{t} = \frac{12.184{,}464}{18.552{,}156} = 0.656{,}768$$

and

$$f_p = \frac{807.4}{1.05958} \cdot \frac{1 + 0.656{,}768 \;\; \dfrac{1.05958}{0.134{,}533}}{1 + 0.656{,}768 \;\; \dfrac{807.4}{1350} \cdot \dfrac{1}{0.134{,}533}}$$

$$f_p = 1199.99$$

The values, p, e/t, B, and A corresponding to the allowable stress of 1200 psi. are taken from the above trial computation, and recorded in table 2.

It now remains to find the moment M or Ne, and the normal load N which acting together on a column will stress it to the full allowable value, f_p given in formula (14). It is proven in <u>Reinforced Concrete Structures</u> by Dean Peabody that the moment,

$$M = \frac{f_p \; R^3}{96k} \left(W + 24 \pi \, npg^2\right) \tag{15}$$

and

$$N = \frac{f_p \; R^2}{6k} \left(V - 3 \pi \, np \cos a_1\right) = \frac{f_p \; R^2}{6k \; (96)} \tag{16}$$

Then

$$\frac{M}{N} = \frac{Ne}{N} = e$$

or

$$e = \frac{\dfrac{f_p \; R^3}{96k} \left(W + 24 \pi \, npg^2\right)}{\dfrac{f_p \; R^2}{6k} \left(V - 3 \pi \, np \cos a_1\right)}$$

and since $$\frac{e}{2R} = \frac{e}{t},$$

$$\frac{e}{t} = \frac{\frac{1}{96}(W \neq 24\,\pi npg^2)}{\frac{1}{96}(32V - 96\,\pi np \cos a_1)} \tag{16a}$$

Now, as in tied columns, let e/t - B/A and

$$B = \frac{W \neq 24\,\pi\, npg^2}{96}$$

$$A = \frac{(32V - 96\,\pi\, np \cos a_1)}{96}$$

Now $$N = \frac{f_p\, R^2}{2k} \cdot \frac{1}{96}\ (32V - 96\,\pi\, np \cos a_1)$$

or $$N = \frac{f_p\, R^2}{2k} \cdot A$$

since $$R = \frac{t}{2}$$

$$N = \frac{f_p\, t^2}{8k} \cdot A$$

$$N = t^2 \cdot \frac{f_p}{8} \cdot \frac{A}{k} \tag{17}$$

Likewise $$M = \frac{f_p\ t^3}{8k} \cdot B$$

$$M = t. \frac{f_p}{8} \cdot \frac{B}{k} \tag{18}$$

From equations (17) and (18) above

$$t = \sqrt{\frac{8Nk}{f_p\ A}} \tag{19}$$

and also $$t = \sqrt[3]{\frac{8Mk}{f_p\ B}}$$

Therefore $$\sqrt{\frac{8Nk}{f_p\ A}} = \sqrt[3]{\frac{8Mk}{f_p\ B}}$$

Then $$\sqrt{N}\sqrt{\frac{8k}{fA}} = \sqrt{M}\sqrt{\frac{8k}{f_c\ B}}$$

and $$\frac{\sqrt[3]{M}}{\sqrt{N}} = \frac{\sqrt{\frac{8k}{f_p\ A}}}{\sqrt[3]{\frac{8k}{f_p\ B}}}$$

also
$$\frac{\sqrt[3]{M}}{\sqrt{N}} = \frac{\left[\frac{8k}{f_p}\right]^{\frac{1}{2}} \frac{1}{\sqrt{A}}}{\left[\frac{8k}{f_p}\right]^{1/3} \frac{1}{\sqrt[3]{B}}}$$

or
$$\frac{\sqrt[3]{M}}{\sqrt{N}} = \left[\frac{8k}{f_p}\right]^{1/6} \frac{\sqrt[3]{B}}{\sqrt{A}} \tag{20}$$

also from equation (17) $t^2 = \dfrac{N}{\dfrac{f_p\ A}{8k}}$, a design formula (21)

The values of $\sqrt[3]{\dfrac{M}{\sqrt{N}}}$ and $\dfrac{f_p\ A}{8k}$ are plotted against per cent steel in chart 2. To design a spiral column for a known moment M and normal load N, the value of $\sqrt[3]{\dfrac{M}{\sqrt{N}}}$ is determined by slide rule and is noted on the curve so marked. The value of p, the per cent steel may then be read from the abscissa directly below. A vertical line passing through p will intersect the curve $\dfrac{f_p\ A}{8k}$. The ordinate of this point read from the right side of the chart gives the value $\dfrac{f_p\ A}{8k}$. Then $t = 2R$ is readily determined from equation (21). This is the design column diameter.

CHAPTER III

PRACTICAL APPLICATIONS OF DIRECT DESIGN OF CONCRETE COLUMNS

Design of a rectangular tied column.--Design an exterior concrete column of rectangular cross section to withstand a normal load of 40 kips together with a bending moment of 100 kip feet.

$$M = Ne$$

$$e = \frac{100(12)}{40} = 30$$

Try $$t = 20$$

Then $$\frac{e}{t} = \frac{30}{20} = 1.5$$

Steel ratio p, (from chart 1) = 0.0158

From formula (3) $$bt = \frac{N}{\frac{1}{2}f_pA}$$

From chart 1 $$\frac{1}{2}f_pA = 112$$

Then $$b = \frac{40{,}000}{112\ (20)} = 17.86$$

and $A_s = .0158(20)(17.86) = 5.68$ sq. in.

Therefore, a practical design would be an 18 x 20 inch column with six number nine bars for vertical reinforcement. This furnishes slightly more concrete and steel than is necessary for the required axial load and bending moment.

A check of the above design by an independent method follows: The steel on the compression side of the column is 2.84 sq. in.

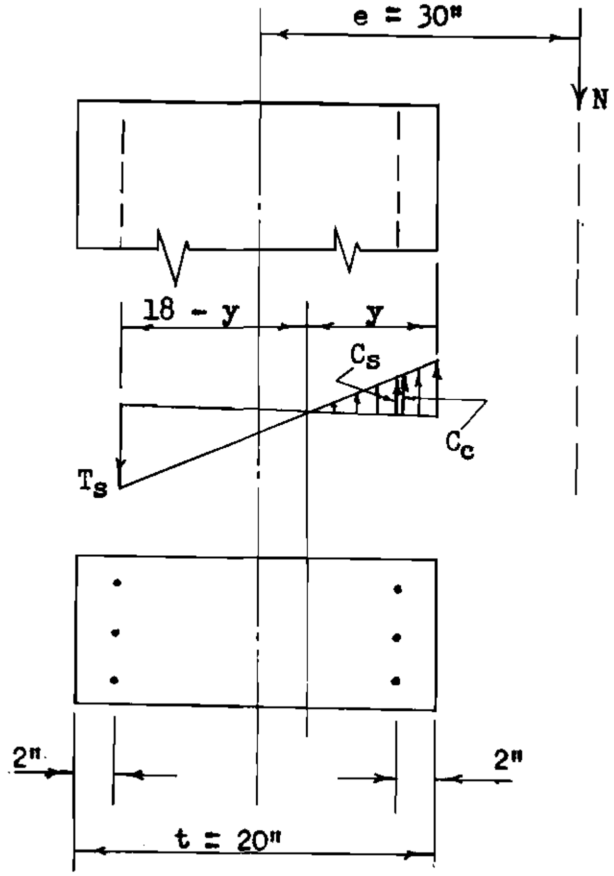

Fig. 4

Tied column section

$$C_c = \tfrac{1}{2}f_c\ (17.86)\ (y) = 8.928f_c\ y$$

$$C_s = (n - 1)\ \frac{(y - 2)}{y}\ f_c A_s$$

$$C_s = 9\ .\ \frac{(y - 2)}{y}\ f_c\ (2.84)$$

$$C_s = 25.60\ .\ \frac{y - 2}{y}\ f_c$$

$$T_s = \frac{f_c}{y}\ (18 - y)\ (10)\ (2.84)$$

$$T_s = 28.44\ \frac{f_c}{y}\ (18 - y)$$

The summation of moments about the line of action of the force N is equal to zero.

$$28.44\ \frac{f_c}{y}\ (18 - y)\ (38) - 25.60\ \frac{(y - 2)}{y}\ f_c \tag{22}$$

$$-8.928f_c\ y \left(20 \neq \frac{y}{3} \right)$$

This is a cubic equation in y and was solved by trial and error. After several trials "y" was found to be 6.846 in inches.

Then $$C_c = 8.928\ (6.846)\ f_c = 61.125f_c$$

$$C_s = 25.60 \frac{(4.846)}{6.846} f_c = 18.118 f_c$$

$$T_s = 28.44 \frac{(11.154)}{6.846} f_c = 46.337 f_c$$

The sum of vertical forces acting on a length ds of column is equal to zero and

$$C_c \neq C_s - T_e = N$$

$$32.906 f_c = 40{,}000$$

$$f_c = 1216 \text{ psi}$$

This would be the actual stress under that load and moment. Interpolating in Table 5 for a value of f_p corresponding to an e/t of 1.5 gives the allowable value of $f_p = 1222$ psi. Formula (7) also gives a value of $f_p = 1222$ psi. This is a very close check between the allowable value of f_p obtained by the American Concrete Institute formula and the actual stress in the column. The charts can only be read to three significant figures.

The actual tensile stress f_s in the steel is,

$$f_s = \frac{T_s}{\frac{A_s}{2}}$$

or

$$f_s = \frac{46.337\ (1216)}{2.84}$$

$$= 19{,}840 \text{ psi}$$

which is just under the allowable 20,000 psi.

The actual stress in the steel on the compression side is $\frac{18.118 f_c}{2.84} = 7{,}757$ psi. This is an excess of compressive steel, however it would be required as tensile steel if the moment should be reversed by a shifting of loads.

The initial design is very near ideal in that the maximum allowable values of f_c and f_s is utilized in the column when it is subjected to the design load and moment.

<u>Design of a round spiral column</u>.--Design a round spiral column using the load and moment on page 24.

$$M = 1200 \text{ kip - inches}$$

$$N = 40 \text{ kips}$$

$$\sqrt[3]{\frac{M}{N}} = \frac{1200}{40} = \frac{10.626}{6.324} = 1.680$$

$$P = 0.0151 \quad \text{(from chart 2)}$$

$$\frac{f_p\ A}{8k} = 0.0665 \text{ (from chart 2)}$$

$$t^2 = \frac{N}{\frac{f_p A}{8k}} \qquad \text{(from formula 21)}$$

$$t = \sqrt{\frac{40}{0.0665}} = 24.526 \text{ inches}$$

$$A_c = \frac{(24.526)}{4} = 472.436 \text{ sq. in.}$$

$$A_s = 0.0151\ (472{,}436) = 7.1338 \text{ sq. in.}$$

Twelve number seven bars should be satisfactory.

$$R = \frac{t}{2} = 12.263 \text{ inches}$$

$$e = \frac{1200}{40} = 30 \text{ inches}$$

$$\frac{e}{t} = \frac{30}{24.526} = 1.223$$

To check this design it is necessary to determine the position of the neutral axis. See Fig. 4. The angle a, assumed to be the same for both the equivalent steel ring and the outside of the column, determines its position. The value of k and therefore 2kR may be readily determined once a, is known. Then since e/t, the moment, and axial load are known it remains to compute the actual maximum stress in both steel and concrete.

From formulae (11) and (15) where $p = 0.0151$, $g = 0.8$ and

$$C_m = W + 24\pi npg^2$$

$$C_n = V - 3\pi np \cos a_1$$

$$\frac{e}{t} = \frac{M}{2NR} = \frac{C_m}{32C_n}$$

Values of a_1 were tested in the above formulae until $a_1 = 72^\circ\ 39'$ was determined as the angle for $e/t = 1.222$. Since e/t in the above column was 1.223, a_1 has been accurately determined.

Now $$k = \frac{1 - \cos a_1}{2} = 0.3509$$

and $$f_c = \frac{96\ kM}{C_m\ R^3}$$

$$f_c = \frac{96\ (0.3509)\ (1,200,000)}{17.012\ (12.263)^3}$$

$$f_c = 1288 \text{ psi. (actual)}$$

The actual stress in the steel is:

$$f_s = nf_c \cdot \frac{d - kt}{kt}$$

$$f_s = 10(1288)\,\frac{22.08 - 8.606}{8.606}$$

$$f_s = 20{,}165 \quad \text{actual}$$

Now that the actual stress has been determined in the concrete it remains to determine if the allowable value in the American Concrete Institute Code has been exceeded. Substituting in formula (14);

$$f_p = \frac{977}{1.1359} \cdot \frac{1 + 1.223.\ \dfrac{1.1359}{.14674}}{1 + 1.223.\ \dfrac{977}{1350} \cdot \dfrac{1}{.14674}}$$

$$f_p = 1280 \text{ psi. (allowable)}$$

This value compared to 1288 psi., the actual value, is well within the limits of accuracy in concrete design.

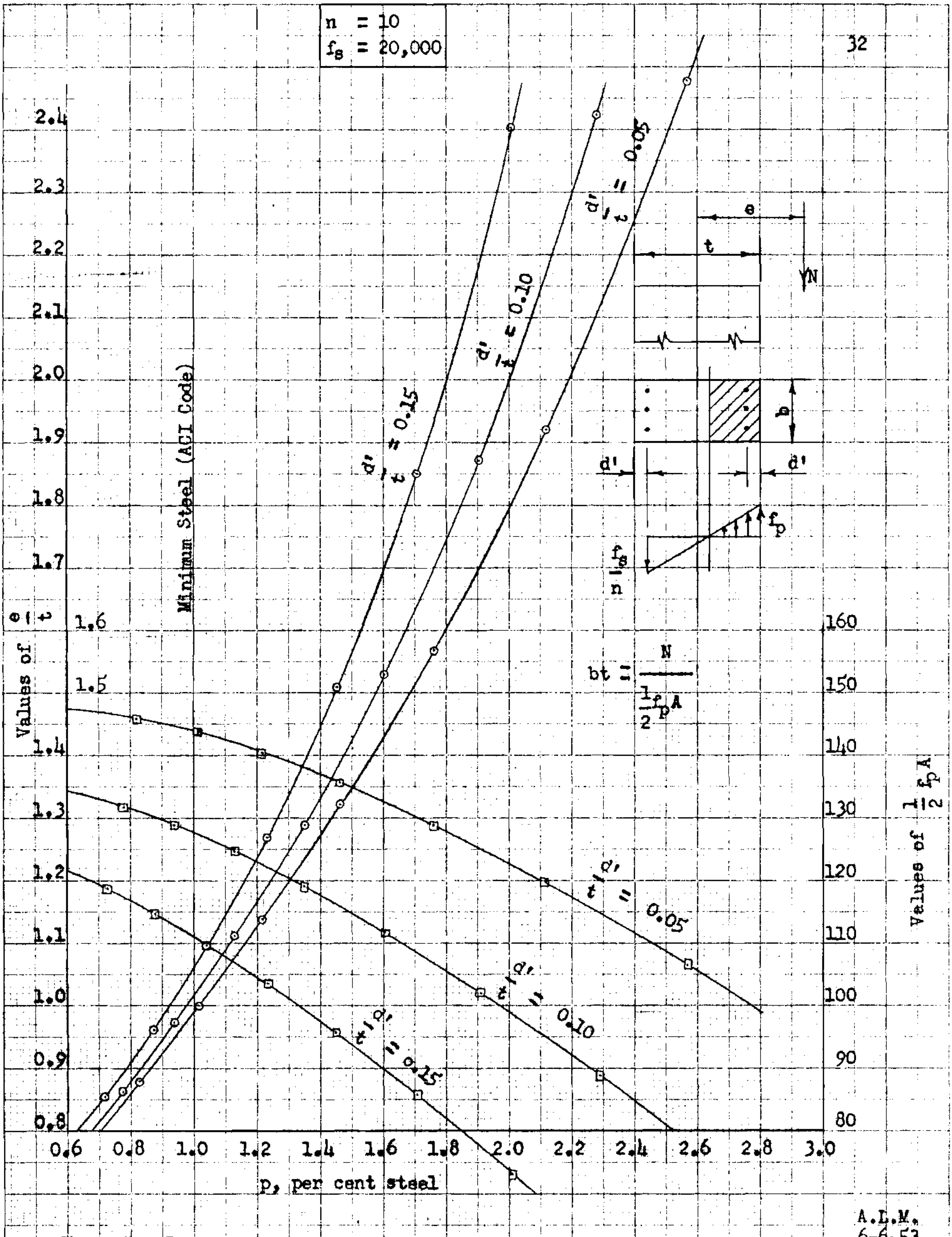

Chart 1. Design Chart for Tied Columns

A.L.M.,
6-6-53

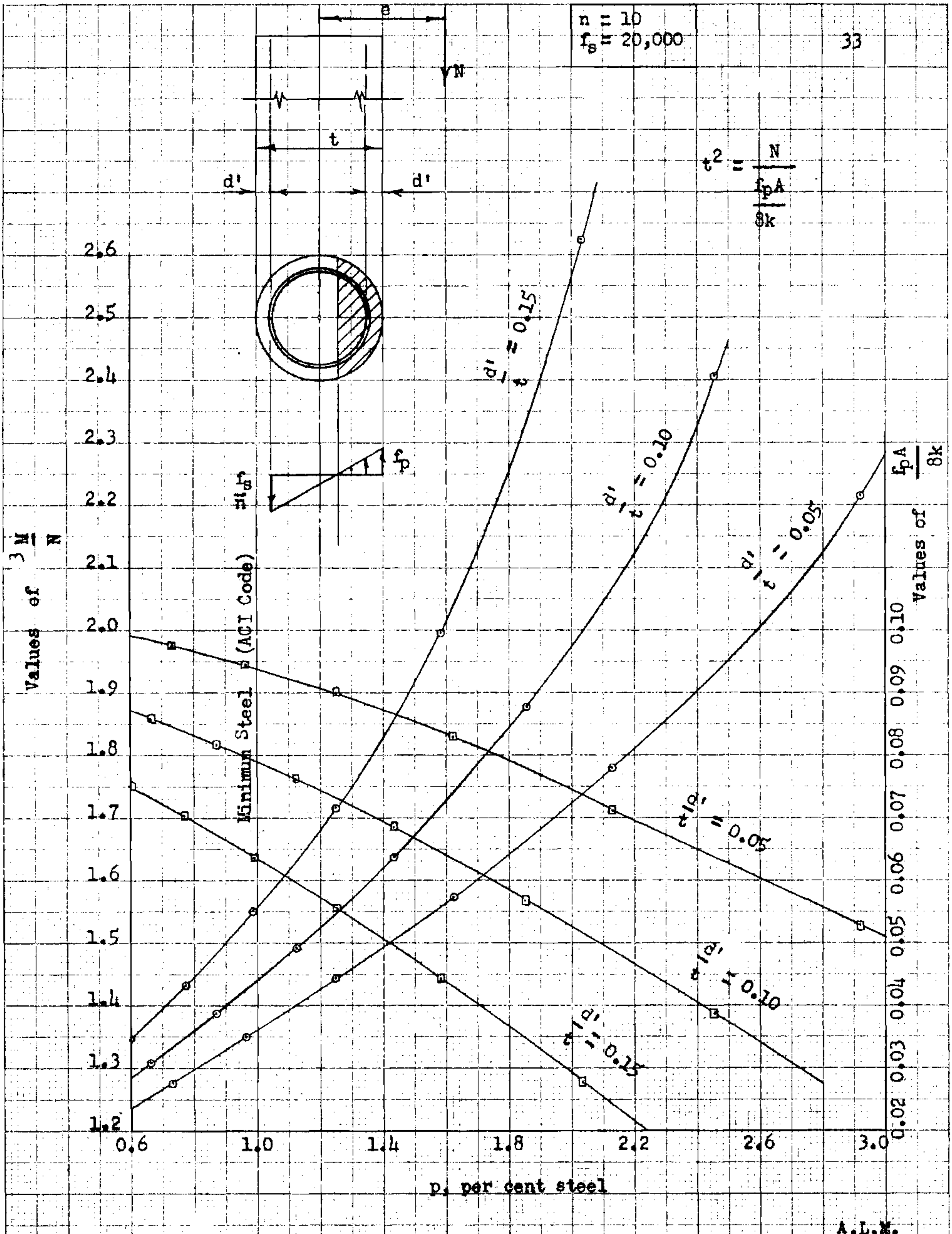

Chart 2. Design Chart for Round Spiral Columns

A.L.M.
6-6-53

CHAPTER IV

CONCLUSIONS

1. Chart 1 makes possible a direct balanced design of a tied column subjected to bending and direct load, where L/D is not greater than ten.

2. Chart 2 makes possible a direct balanced design of a round spiral column subjected to bending and direct load, where L/D is not greater than ten.

3. Charts 1 and 2 are limited to columns with sufficient bending to cause tensile cracks in the concrete.

4. The maximum value of $f_c = 1350$ pounds per square inch for the concrete used throughout this treatise is never quite reached when using the above charts.

5. If the value e/t for any column is known and falls within the range of values and conditions set forth, the allowable stress, f_p, may be found by interpolation from Table 1 through Table 6.

CHAPTER V

RECOMMENDATIONS

1. It would be very beneficial to the designer if charts were prepared for other strengths of concrete than 3000 pounds per square inch used here.

2. Since for architectural reasons columns sizes are not often set by the structural engineer, it would be of paramount interest to the designer to be able to use charts 1 and 2 for unbalanced designs.

3. Charts might also be made for long columns with L/D equal to or greater than 10.

APPENDIX

Table 1. Tabulations for Solution of Formulae (5) and (7) for Tied Columns ($\frac{d'}{t} = 0.05$)

1	2	3	4	5	6	7
f_p	$k = \frac{0.95 f_p}{f_p + 2000}$	$\frac{k}{2}$	$\frac{k^2}{3}$	$\frac{k}{2} - \frac{k^2}{3}$	$9.05 - k$	$\frac{0.45(9.05-k)}{k}$
1125	0.342	0.171	0.038,988	0.132,012	8.708	11.45789
1150	0.346,825	0.173,412	0.040,096	0.133,316	8.703,175	11.29223
1175	0.351,575	0.175,788	0.041,202	0.134,586	8.698,425	11.13359
1200	0.356,250	0.178,125	0.042,305	0.135,820	8.693,750	10.98158
1225	0.360,853	0.180,427	0.043,405	0.137,022	8.689,147	10.83576
1250	0.365,384	0.182,692	0.044,502	0.138,190	8.684,616	10.69581
1275	0.369,847	0.184,924	0.045,596	0.139,328	8.680,153	10.56131

1	8	9	10	11	12	13
f_p	19k	$\frac{(19k - 9.95)}{k}$	p	e/t	A = #2 + p (#9) #2 + #10(#9)	$\frac{1}{2} f_p A$ $\frac{1}{2}$(#1)(#12)
1125	6.498	-10.093,567	0.00827	0.877	0.258,526	145.42
1150	6.589,675	- 9.688,820	0.01009	0.993	0.249,065	143.21
1175	6.679,925	- 9.301,216	0.01219	1.135	0.238,193	139.94
1200	6.768,750	- 8.929,825	0.01467	1.318	0.225,249	135.15
1225	6.856,207	- 8.573,555	0.01762	1.563	0.209,787	128.49
1250	6.942,296	- 8.231,625	0.02123	1.916	0.190,627	119.14
1275	7.027,093	- 7.903,017	0.02575	2.472	0.166,344	106.04

$n - k \frac{d'}{t}(2n-1) = 9.05 - k$ $\quad$ $\frac{1}{2} - \frac{d'}{t} = 0.45$ $\quad$ $k\,(2n-1) - n + \frac{d'}{t} = 19k - 9.95$

Table 2. Tabulations for Solution of Formulae (5) and (7) for Tied Columns ($\frac{d'}{t} = 0.10$)

1	2	3	4	5	6	7
f_p	$k = \frac{0.9\, f_p}{f_p + 2000}$	$\frac{k}{2}$	$\frac{k^2}{3}$	$\frac{k}{2} - \frac{k^2}{3}$ #3 - #4	$8.1 - k$	$\frac{0.4(8.1-k)}{k}$
1125	0.324	0.162	0.034,992	0.127,008	7.776	9.600,000
1150	0.328,571	0.164,286	0.035,986	0.128,300	7.771,429	9.460,882
1175	0.333,071	0.166,536	0.036,979	0.129,557	7.766,929	9.327,656
1200	0.337.5	0.168,750	0.037,969	0.130,781	7.762,500	9.200,000
1225	0.341,861	0.170,930	0.038,956	0.131,974	7.758,139	9.077,536
1250	0.346,154	0.173,077	0.039,941	0.133,136	7.753,846	8.959,996
1275	0.350,382	0.175,191	0.040,923	0.134,268	7.749,618	8.847,050

1	8	9	10	11	12	13
f_p	19k	$\frac{19k - 9.9}{k}$	f_p	e/t	A = #2+p(#9) or #2 + #10(#9)	$\frac{1}{2}f_pA$ $\frac{1}{2}$(#1)(#12)
1125	6.156,000	-11.555,556	0.00777	0.861	0.234,233	131.7
1150	6.242,849	-11.130,474	0.00942	0.972	0.223,722	128.6
1175	6.328,349	-10.723,392	0.01131	1.110	0.211,789	124.4
1200	6.412,500	-10.333,333	0.01350	1.287	0.198	118.8
1225	6.495,359	- 9.959,138	0.01605	1.526	0.182,017	111.5
1250	6.576,926	- 9.599,987	0.01910	1.869	0.162,794	101.7
1275	6.657,258	- 9.254,876	0.02285	2.422	0.138,908	88.6

$\frac{1}{2} - \frac{d'}{t} = 0.4$ $\quad$ $n-k-\frac{d'}{t}(2n-1) = 8.1-k$ $\quad$ $k(2n-1)-n+\frac{d'}{t} = 19k - 9.9$

Table 3. Tabulations for Solution of Formulae (5) and (7) for Tied Columns ($\frac{d'}{t} = 0.15$)

1	2	3	4	5	6	7
f_p	$k = \frac{0.85f_p}{f_p \neq 2000}$	$\frac{k}{2}$	$\frac{k^2}{3}$	$\frac{k}{2} - \frac{k^2}{3}$	$7.15 - k$	$\frac{0.35\ (7.15 - k)}{k}$
1125	0.306	0.153,000	0.031,212	0.121,788	6.844	7.82810
1150	0.310,317	0.155,159	0.032,099	0.123,060	6.83968	7.71433
1175	0.314,567	0.157,284	0.032,984	0.124,300	6.83543	7.60538
1200	0.318,750	0.159,375	0.033,867	0.125,508	6.83125	7.50098
1225	0.322,868	0.161,434	0.034,748	0.126,686	6.82713	7.40084
1250	0.326,923	0.163,462	0.035,626	0.127,836	6.82308	7.30471
1275	0.330,916	0.165,458	0.036,502	0.128,956	6.81908	7.21234

1	8	9	10	11	12	13
f_p	19k	$\frac{19k - 9.85}{k}$	p	e/t	$A = \frac{\#2 \neq p(\#9)}{\#2 \neq \#10\ (\#9)}$	$\frac{\frac{1}{2} f_p A}{\frac{1}{2}(1)(12)}$
1125	5.81400	13.1895	0.00727	0.850	0.210,112	118.18
1150	5.89602	12.7417	0.00876	0.959	0.198,700	114.25
1175	5.97677	12.3129	0.01043	1.094	0.186,143	109.36
1200	6.05625	11.9020	0.01234	1.269	0.171,879	103.13
1225	6.13449	11.5078	0.01453	1.505	0.155,660	95.34
1250	6.21154	11.1294	0.01709	1.848	0.136,722	85.45
1275	6.28740	10.7659	0.02013	2.401	0.114,198	72.80

$n - k - \frac{d'}{t}(2n - 1) = (7.15 - k)$ $\qquad \frac{1}{2} - \frac{d'}{t} = 0.35$ $\qquad k(2n - 1) - n \neq - \frac{d'}{t}(19k - 985)$

Table 4. Tabulations for Solution of Formulae (13) and (14) for Circular Spiral Columns ($\frac{d'}{t} = 0.05$)

1	2	3	4	5	6	7
f_p	k	2k	Cos a' 1 - 2k	Degrees	Radians	Sin a'
1200	0.356,250	0.712,500	0.287,500	73.291,667	1.279,181	0.957,781
1225	0.360,853	0.721,706	0.278,294	73.841,585	1.288,585	0.960,496
1250	0.365,384	0.730,768	0.269,232	74.381,417	1.298,201	0.963,075
1275	0.369,847	0.739,694	0.260,306	74.911,780	1.307,457	0.965,526
1300	0.374,242	0.748,484	0.251,516	75.432,751	1.316,550	0.967,853
1325	0.378,571	0.757,142	0.242,858	75.944,724	1.325,485	0.970,062

1	8	9	10	11	12
f_p	(#4)(#7) Sin a' Cos a'	#8 - #6 Sin a' Cos a' - a'	3(#4)(#9) (3 Cos a')(Sin a' Cos a' - 1)	$2(\#7)^3$ $2 \sin^3 a'$	10 + 11 V
1200	0.275,362	-1.003,819	-0.865,794	1.757,229	0.891,435
1225	0.267,300	-1.021,479	-0.852,814	1.772,216	0.919,402
1250	0.259,291	-1.038,910	-0.839,123	1.786,532	0.947,409
1275	0.251,332	-1.056,125	-0.824,747	1.800,205	0.975,458
1300	0.243,431	-1.073,119	-0.809,720	1.813,253	1.003,533
1325	0.235,587	-1.089,898	-0.794,071	1.825,695	1.031,624

Table 4 (Cont'd)

1	13	14	15	16	17	18
f_p	4(#5) $4a_1$	-3 Sin (#13) -3 Sin $4a_1$	12(#6) $12a_1$	$-32(\#8)(\#7)^2$ $-32 \sin^3 a_1 \cos a_1$	14+15+16 W	32(#12) 32V
1200	66° 50' 00"	2.758,093	15.350,172	-8.083,253	10.025,012	28.525,920
1225	64° 38' 01.2"	2.710,762	15.465,348	-7.891,144	10.284,966	29.420,864
1250	62° 28' 27.6"	2.660,412	15.578,412	-7.695,873	10.542,951	30.317,088
1275	60° 21' 10.4"	2.607,266	15.689,484	-7.497,661	10.799,089	31.214,656
1300	58° 16' 8.4"	2.551,580	15.798,600	-7.297,007	11.053,173	32.113,056
1325	56° 13' 16"	2.493,568	15.905,820	-7.094,147	11.305,241	33.011,968

1	19	20	21	22	23	24
f_p	$96\pi n(\#4)$ $96\pi n \cos a_1$	p	$\frac{e}{t}$	$\frac{(\#17 + 610.726)(\#20)}{96}$ B	$\frac{\#18 - \#19(\#20)}{96}$ A	$\sqrt[3]{B}$
1200	867.080	0.00729	0.652	0.150,804	0.231,301	0.532,277
1225	839.315	0.00962	0.757	0.168,335	0.222,361	0.552,151
1250	811.985	0.01251	0.902	0.189,408	0.209,991	0.574,292
1275	785.064	0.01623	1.121	0.215,741	0.192,428	0.599,761
1300	758.554	0.02132	1.510	0.250,769	0.166,049	0.630,606
1325	732.442	0.02921	2.509	0.303,589	0.121,014	0.672,092

E

Table 4 (Cont'd)

1	25	26	27	28	29
f_p	$\sqrt{A}$	$\frac{8k}{f_p}$ (1000) (f_p in ksi.)	$\sqrt[6]{\frac{8k}{f_p}}$ (1000) (f_p in ksi.)	$\left[\frac{8k}{f_p}\right]^{\frac{1}{6}}$ $\frac{\sqrt[3]{B}}{\sqrt{A}}$	$\frac{f_pA}{8k}$
1200	0.480,938	2.375	1.155	1.278	0.0974
1225	0.471,552	2.357	1.154	1.351	0.0944
1250	0.458,248	2.338	1.152	1.444	0.0898
1275	0.438,666	2.321	1.151	1.573	0.0829
1300	0.407,491	2.303	1.149	1.778	0.0721
1325	0.347,871	2.286	1.148	2.217	0.0529

Table 5. Tabulations for Solution of Formulae (13) and (14) for Circular Spiral Columns ($\frac{d'}{t} = 0.10$)

1	2	3	4	5	6	7
f_p	k	2k	Cos a_1 = 1 - 2k	Degrees	Radians	Sin a_1
1200	0.337,500	0.675,000	0.325,000	71.034,444	1.239,785	0.945,714
1225	0.341,861	0.683,722	0.316,278	71.562,030	1.248,993	0.948,667
1250	0.346,154	0.692,308	0.307,692	72.079,805	1.258,030	0.951,486
1275	0.350,823	0.700,764	0.299,236	72.588,278	1.266,904	0.954,179
1300	0.354,545	0.709,909	0.290,910	73.087,556	1.275,618	0.956,750
1325	0.358,647	0.717,294	0.282,706	73.578,222	1.284,182	0.959,207
1350	0.362,687	0.725,374	0.274,626	74.060,278	1.292,596	0.961,551

1	8	9	10	11	12	13
f_p	Sin a_1 Cos a_1	#8 - #6 Sin a_1 Cos $a_1 - a_1$	3(#4)(#9) 3 Cos a_1 x (Sin a_1 Cos $a_1 - a_1$)	$2(\#7)^3_3$ 2 Sin^3 a'	10 ≠ 11 V	4 (#5) 4 a_1
1200	0.307,357	-0.932,428	-0.909,117	1.691,646	0.782,529	-75 51' 44"
1225	0.300,043	-0.948,950	-0.900,396	1.707,542	0.807,146	-73 45' 08"
1250	0.292,765	-0.965,265	-0.891,013	1.722,810	0.831,797	-71 40' 52"
1275	0.285,525	-0.981,379	-0.880,992	1.737,478	0.856,486	-69 38' 49"
1300	0.278,328	-0.997,290	-0.870,365	1.751,562	0.881,197	-67 38' 59.2"
1325	0.271,174	-1.013,008	-0.859,150	1.765,909	0.905,940	-65 41' 136"
1350	0.264,067	-1.028,529	-0.847,382	1.778,062	0.930,680	-63 45' 32"

Table 5 (Cont'd)

1	14	15	16	17	18	19
f_p	-3 Sin (#13) -3 Sin 4 a_1	+ 12 (#6) + 12 a_1	-32 (#8)$(\#7)^2$ -32 $\text{Sin}^3 a_1$ Cos a_1	14+15+16 W	32(#12) 32V	$96\pi n$ (#4) $96\pi n$ Cos a_1
1200	+2.909,133	14.877,420	-8.796,557	8.989,996	25.040,928	980.177
1225	+2.880,183	14.987,916	-8.640,942	9.227,157	25.828,672	953.872
1250	+2.847,966	15.096,360	-8.481,525	9.462,801	26.617,504	927.977
1275	+2.812,701	15.202,848	-8.318,673	9.696,876	27.407,552	902.475
1300	+2.774,631	15.307,416	-8.152,745	9.929,302	28.198,304	877.364
1325	+2.733,933	15.410,184	-7.984,040	10.160,077	28.990,080	852.621
1350	+2.690,823	15,511,152	-7.812,837	10.389,138	29.781,760	828.253

1	20	21	22	23	24
f_p	p	$\frac{e}{t}$	$\frac{\#17 + 482.548(\#20)}{96}$ B	$\frac{\#18 - \#19(\#20)}{96}$ A	$\sqrt[3]{B}$
1200	0.00662	0.657	0.126.922	0.193,252	0.502,549
1225	0.00870	0.766	0.139,847	0.182,604	0.519,060
1250	0.01122	0.918	0.154,969	0.168,812	0.537,133
1275	0.01437	1.152	0.173,240	0.150,406	0.557,463
1300	0.01854	1.582	0.196,622	0.124,291	0.581,493
1325	0.02459	2.745	0.229,437	0.083,585	0.612,192
1350					

Table 5 (Cont'd)

1	25	26	27	28	29
f_p	$\sqrt{A}$	$\frac{8k}{f_p}(1000)$ (f_p in ksi.)	$\sqrt[6]{\frac{8k}{f_p}(1000)}$ (f_p in ksi.)	$\left[\frac{8k}{f_p}\right]^{\frac{1}{6}} \frac{\sqrt[3]{B}}{\sqrt{A}}$	$\frac{f_p A}{8k}$
1200	0.439,604	2.250	1.145	1.309	0.0859
1225	0.427,322	2.232	1.143	1.389	0.0818
1250	0.410,867	2.215	1.142	1.493	0.0762
1275	0.387,822	2.198	1.140	1.639	0.0684
1300	0.352,550	2.182	1.139	1.878	0.0570
1325	0.289,110	2.165	1.137	2.409	0.0386
1350					

Table 6. Tabulations for Solution of Formulae (13) and (14) for Circular Spiral Columns ($\frac{d'}{t} = 0.15$)

1	2	3	4	5	6	7
f_p	k	2k	Cos a_1 1 - 2k	Degrees	Radians	Sin a_1
1200	0.318,750	0.637,500	0.362,500	68.746,195	1.199,847	0.931,984
1225	0.322,868	0.645,736	0.354,264	69.251,667	1.208,670	0.935,146
1250	0.326,923	0.653,846	0.346,154	69.747,750	1.217,328	0.938,178
1275	0.330,916	0.661,832	0.338,168	70.234,695	1.225,827	0.941,086
1300	0.334,848	0.669,696	0.330,304	70.712,778	1.234,171	0.943,875
1325	0.338,722	0.677,444	0.322,556	71.182,417	1.242,368	0.946,550

1	8	9	10	11	12	13
f_p	Sin a_1 Cos a_1	#8 - #6	3(#4)(#9) (3 Cos a_1) (Sin a_1 Cos $a_1 - a_1$)	2(#7)	10 + 11 V	4(#5) 4 a_1
1200	0.337,844	-0.862,000	-0.937,428	1.619,030	0.681,602	274° 59' 05.2"
1225	0.331,288	-0.877,382	-0.932,475	1.635,564	0.703,089	277° 00' 24"
1250	0.324,754	-0.892,574	-0.926,904	1.651,526	0.724,622	278° 59' 27.6"
1275	0.318,245	-0.907,582	-0.920,746	1.666,930	0.746,184	280° 56' 19.6"
1300	0.311,766	-0.922,405	-0.914,022	1.681,794	0.767,772	282° 51' 04"
1325	0.305,315	-0.937,053	-0.906,756	1.696,138	0.789,382	284° 43' 46.8"

Table 6 (Cont'd)

1	14	15	16	17	18	19
f_p	-3 Sin (#13) -3 Sin $4a_1$	12 (#6) $12a_1$	$-32(\#8)(\#7)^2$ $-32\ \mathrm{Sin}^3 a_1\ \mathrm{Cos}\ a_1$	14+15+16 W	32(#12) 32V	$96\pi n(\#4)$ $96\pi n\ \mathrm{Cos}\ a_1$
1200	+2.988,653	14.398,164	-9.390,375	7.996,442	21.811,264	1093.274
1225	+2.977,596	14.504,040	-9.270,733	8.210.903	22.498,848	1068.435
1250	+2.963,139	14.607,936	-9.146,918	8.424,157	23.187,904	1043.976
1275	+2.945,492	14.709,924	-9.019,239	8.636,177	23.877,888	1019.891
1300	+2.924,854	14.810,052	-8.888,068	8.846,838	24.568,704	996.173
1325	+2.901,409	14.908,416	-8.753,576	9.056,249	25.260,224	972.806

1	20	21	22	23	24	25
f_p	p	$\frac{e}{t}$	$\frac{\#17 + 369.451(\#20)}{96}$ B	$\frac{\#18 - \#19(\#20)}{96}$ A	$\sqrt[3]{B}$	$\sqrt{A}$
1200	0.00595	0.666	0.106,194	0.159,441	0.473,551	0.399,300
1225	0.00776	0.780	0.115,394	0.147,998	0.486,850	0.384,705
1250	0.00992	0.942	0.125,928	0.133,663	0.501,234	0.365,600
1275	0.01254	1.197	0.138,220	0.115,505	0.517,039	0.334,860
1300	0.01585	1.675	0.153,152	0.091,452	0.535,026	0.302,410
1325	0.02033	3.021	0.172,575	0.057,115	0.556,749	0.238,988

Table 6 (Cont'd)

1	26	27	28	29
f_p	$\frac{8k}{f_p}(1000)$ (f_p in ksi.)	$\sqrt[6]{\frac{8k}{f_p}(1000)}$ (f_p in ksi.)	$\left[\frac{8k}{f_p}\right]^{\frac{1}{6}}$ $\frac{\sqrt[3]{B}}{\sqrt{A}}$	$\frac{f_p A}{8k}$
1200	2.125	1.134	1.345	0.0750
1225	2.109	1.132	1.433	0.0702
1250	2.092	1.131	1.551	0.0639
1275	2.076	1.129	1.718	0.0556
1300	2.061	1.128	1.996	0.0444
1325	2.045	1.127	2.625	0.0279

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